Possible η-Mesic ^{3}He States within the Finite Rank Approximation

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Abstract

We extend the method of time delay proposed by Eisenbud and Wigner, to search for unstable states formed by η mesons and the ^{3}He nucleus. Using few body equations to describe η-^{3}He elastic scattering, we predict resonances and unstable bound states within different models of the ηN interaction. The $\eta \, ^3$He states predicted within this novel approach are in agreement with the recent claim of the evidence of η-mesic ^{3}He made by the TAPS collaboration.

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A claim of positive evidence for the existence of the long sought after η-mesic nuclei was made last year by the TAPS collaboration [1] on the basis of their investigations on the photoproduction reaction, $\gamma \, ^3$He $\to \eta X$. This indeed comes as a big step in the understanding of many nuclear and particle physics related problems of fundamental interest. We list here the connection of η-physics to a few of these important issues: (i) the η-N interaction is dominated by the coupling to the $N^*(1535)$ resonance and hence, studies of the η meson in the nuclear medium could yield information on the $\eta N N^*$ coupling constant. (ii) The isospin and charge symmetry breaking (CSB) in quantum chromodynamics (QCD) arises due to the difference in the up and down quark masses due to their electromagnetic interaction. The observation of the π-η-η' mixing provides one of the best possibilities to study directly the CSB effects [2]. (iii) The relative cross sections for the reactions ηp, $\eta' p \to \eta p$, $\eta' p$ and $\pi^- p \to \eta n$, $\eta' n$, provide a sensitive test for the presence of a strange-antistrange ($s\bar{s}$) component in the nucleon's wave function [3]. Obviously then, this evidence of an η-mesic ^{3}He increased the interest in this field which started about a decade ago [4, 5]. We therefore found it timely to search for the η-mesic resonances using the method of time delay proposed by Eisenbud and Wigner [6] about half a century ago, but revived only recently to successfully characterize the meson and baryon resonances [7, 8]. From our experience in [7], we found that with this method (which is mentioned in text books and literature as a necessary condition for the existence of a resonance) we not only confirmed the existing resonances, but also found new ones. Our finding of the exotic pentaquark resonance around 1540

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MeV from K^+N data [8] was indeed confirmed by several recent experiments (listed in [8]).

The time delay in $\eta\,^3$He elastic scattering is evaluated using few-body equations for the $\eta\,^3$He system involving inputs from different models of the elementary ηN interaction. In an earlier work [9], the near threshold data on the $pd \rightarrow {}^3$He η reaction were well reproduced using the same few body equations to describe the $\eta\,^3$He final state interaction. In what follows, the method of time delay is briefly introduced, followed by an example which demonstrates the usefulness of this concept even at negative energies.

Back in the early fifties, Eisenbud and Wigner related the energy derivative of the phase shift to the time delay in scattering as, $\Delta t(E) = 2\,\hbar d\delta/dE$. A more useful definition, namely, the time delay matrix was given by Eisenbud and elaborated by Smith [10]. An element of this matrix, Δt_{ij}, which is the time delay in the emergence of a particle in the jth channel after being injected in the ith channel is given by,

$$\Delta t_{ij} = \Re e\left[-i\hbar(S_{ij})^{-1}\frac{dS_{ij}}{dE}\right],\tag{1}$$

where S_{ij} is an element of the corresponding S-matrix. In an eigenphase formulation of the S-matrix, one can easily see that the time delay as defined above in (1) is the energy derivative of the phase shift. Alternatively, writing the S-matrix in terms of the T-matrix as, $\mathbf{S} = 1 + 2\,i\,\mathbf{T}$, one can evaluate time delay in terms of the T-matrix. The time delay in elastic scattering, i.e. Δt_{ii}, is given in terms of the T-matrix as,

$$S_{ii}^* \, S_{ii}\, \Delta t_{ii}(E) \,=\, 2\,\hbar\,\left[\Re e\left(\frac{dT_{ii}}{dE}\right) + 2\,\Re eT_{ii}\,\Im m\left(\frac{dT_{ii}}{dE}\right) - 2\,\Im mT_{ii}\,\Re e\left(\frac{dT_{ii}}{dE}\right)\right],\tag{2}$$

where $\mathbf{T}$ is the complex T-matrix such that, $T_{kj} \,=\, \Re eT_{kj} \,+\, i\,\Im mT_{kj}$. The above relations were put to a test in [7] to characterize the hadron resonances occurring in meson-nucleon and meson-meson elastic scattering. The energy distribution of the time delay evaluated in these works, nicely displayed the known N and Δ excitations and meson resonances like the ρ, scalars (f_0) and strange K^*'s found in $K\pi$ scattering, in addition to confirming some old claims of exotic states. Evidence for the low-lying exotic pentaquark was also found in addition to several other Z^*'s from the time delay in K^+N scattering.

We shall now demonstrate that the time delay concept which has been so useful in characterizing resonances, can also be extended to the search of bound as well as unstable bound states. The latter signify states with a negative binding energy but having a finite lifetime. These are sometimes called as "quasibound" states in literature. We consider the example of an S-matrix for the $n - p$ system written for the case of a square well potential which has the right parameters to produce the binding energy of the deuteron. The S matrix in this case, as a function of l is given as,

$$S_l = -\frac{\alpha h_l^{(2)'}(\alpha)j_l(\beta) - \beta h_l^{(2)}(\alpha)j_l{}'(\beta)}{\alpha h_l^{(1)'}(\alpha)j_l(\beta) - \beta h_l^{(1)}(\alpha)j_l{}'(\beta)}\tag{3}$$
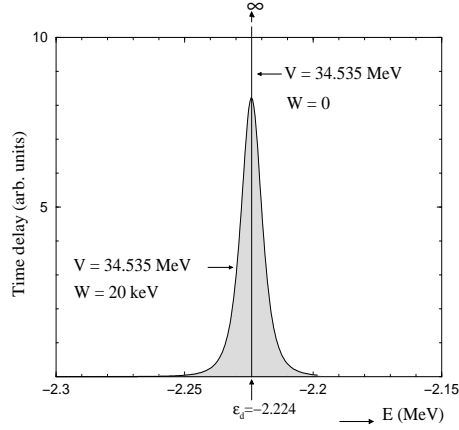
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Figure 1: The theoretical delay time in np elastic scattering as a function of the energy $E = \sqrt{s} - m_n - m_p$, where $\sqrt{s}$ is the total energy available in the np centre of mass system. The sharp spike at $E = -2.224$ MeV corresponding to the deuteron binding energy, indicates the infinite time delay due to the formation of this np bound state. The shaded distribution corresponds to the time delay in the case of a fictitious unstable bound state.

where j_l, $h_l^{(1)}$ and $h_l^{(2)}$ are the spherical Bessel and Hankel functions of the first and second kind respectively. $\alpha = kR$ and $\beta = (\alpha^2 - 2\mu U R^2/\hbar^2)^{1/2}$ where the potential U is given by $-U = V + iW$, with $U(r) = U\,\theta(R-r)$ and R the width of the potential well. A similar square well potential was used by Morimatsu and Yazaki while locating the "unstable bound states" of Σ-hypernuclei as second quadrant poles and by J. Fraxedas and J. Sesma using the time delay method [11].

We evaluate the time delay in $n - p$ scattering at negative energies using the above S-matrix with $\alpha = ikR$ (hence $E = -k^2/2\mu$) and $l = 0$. If we plug in the appropriate parameters for an $n - p$ square well potential, $V = 34.6$ MeV and $R = 2.07$ fm, the time delay plot (as in Fig. 1) shows a sharp spike exactly at the binding energy of the deuteron ($E = -2.224$ MeV). If we add a small imaginary part to the potential, then this state develops into a Breit-Wigner kind of distribution centered around the binding energy of the deuteron (a fictitious "unstable bound state" of the $n - p$ system around $E = -2.224$ MeV). We can thus see that time delay is well-defined at negative energies and does reveal the bound and "unstable bound states" with positive peaks at negative energies. At this point, we may add that in contrast to the infinite time delay that occurs in the case of bound states, the delay time for virtual states (also called "antibound" states) is $-\infty$. Below, we describe our approach to construct the complex T-matrix for $\eta\,^3$He elastic scattering which is required for the calculation of time delay.

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The $\eta \, ^3$He transition matrix is evaluated using few body equations solved within a Finite Rank Approximation (FRA) approach, which means that the ^{3}He nucleus in $\eta \, ^3$He elastic scattering remains in its ground state, in the intermediate state. Since the η-mesic bound states and resonances are basically low energy phenomena, it seems justified to use the FRA for calculations of the present work. Since at low energies, the ηN interaction is dominated by the S_{11} resonance, we restrict ourselves to the study of the s-wave unstable states. The $\eta \, ^3$He t-matrix in this approach is written as [12, 13],

$$
\begin{aligned}
t_{\eta A}(\vec{k'}, \vec{k}\,; z) \;=\; & < \vec{k'}\,; \psi_0 \,|\, t^0(z) \,|\, \vec{k}\,; \psi_0 > + \\
& \varepsilon \int \frac{d\vec{k''}}{(2\pi)^3} \frac{< \vec{k'}\,; \psi_0 \,|\, t^0(z) \,|\, \vec{k''}\,; \psi_0 >}{(z - \frac{k''^2}{2\mu})(z - \varepsilon - \frac{k''^2}{2\mu})} t_{\eta A}(\vec{k''}, \vec{k}\,; z)\,,
\end{aligned}
\tag{4}
$$

where $z = E - |\varepsilon| + i0$. E is the energy associated with ηA relative motion, ε is the binding energy of the nucleus, ψ_0 is the nuclear wave function and μ is the reduced mass of the ηA system. The matrix elements for t^0 are given as,

$$
< \vec{k'}\,; \psi_0 \,|\, t^0(z) \,|\, \vec{k}\,; \psi_0 > = \int d\vec{r}\,|\,\psi_0(\vec{r})\,|^2 \, t^0\,(\vec{k'}, \vec{k}\,; \vec{r}; z)
\tag{5}
$$

where,

$$
t^0\,(\vec{k'}, \vec{k}\,; \vec{r}; z) = \sum_{i=1}^{A} t_i^0\,(\vec{k'}, \vec{k}\,; \vec{r_i}\,; z)\,.
\tag{6}
$$

t_i^0 is the t-matrix for the scattering of the η-meson from the ith nucleon in the nucleus, with the rescattering from the other (A-1) nucleons included. It is given as,

$$
t_i^0\,(\vec{k'}, \vec{k}\,; \vec{r_i}\,; z) = t_i^{\eta N}(\vec{k'}, \vec{k}\,; \vec{r_i}\,; z) + \int \frac{d\vec{k''}}{(2\pi)^3} \frac{t_i^{\eta N}(\vec{k'}, \vec{k''}\,; \vec{r_i}\,; z)}{z - \frac{k''^2}{2\mu}} \sum_{j \neq i} t_j^0(\vec{k''}, \vec{k}\,; \vec{r_j}\,; z)\,.
\tag{7}
$$

The t-matrix for elementary η-nucleon scattering, $t_i^{\eta N}$, is written in terms of the two body ηN matrix $t_{\eta N \to \eta N}$ as,

$$
t_i^{\eta N}(\vec{k'}, \vec{k}\,; \vec{r_i}\,; z) = t_{\eta N \to \eta N}(\vec{k'}, \vec{k}\,; z)\, exp[\,i(\vec{k} - \vec{k'}\,) \cdot \vec{r_i}\,]\,.
\tag{8}
$$

Since there exists a lot of uncertainty in the knowledge of the η-nucleon interaction, we use three different prescriptions [4, 14, 15] of the coupled channel η-N t-matrix, $t_{\eta N \to \eta N}$, leading to different values of the ηN scattering length.

Before proceeding further, we briefly list the general features of the present approach, the approximations made and their validity in the context of the present calculations:

(i) The main idea of the FRA consists of separating the motion of the projectile and the internal motion of the nucleons inside the nucleus. The total Hamiltonian is split accordingly and the approximation in solving these equations as

mentioned above is to restrict the spectral decomposition of the nuclear Hamiltonian to the ground state. This, however, limits the applicability of the present approach to energies below the break up threshold of the ^{3}He nucleus.

(ii) The operator t^0 describes the scattering of the η meson from nucleons fixed in their space position within the nucleus. However, it differs from the usual fixed center t-matrices as t^0 is taken off the energy shell and involves the motion of the η meson with respect to the center of mass of the target. The present scheme should not be confused with an optical potential approach which involves an impulse approximation and the omission of higher order rescattering terms. Besides, the t-matrix is evaluated at $z = E - |\varepsilon| + i0$ in contrast to the usual fixed scatterer approximations which do not involve the binding energy term.

(iii) the dominance of the $N^* - S_{11}$ resonance which decays with a similar branching ratio to both the πN and ηN channels implies that the transitions $\eta N \to \pi N$ must be taken into account. In the present work we use coupled channel t-matrices for the elementary η-nucleon interaction which include fully the effect of the πN and ηN channel on each other. The complex self energy terms in the $\eta N \to \eta N$ t-matrix take care of the intermediate off shell as well as on shell πN loops. We neglect however, transitions involving the production of an η meson on a second nucleon by a pion produced on the first nucleon. For example, in the present three nucleon case, we neglect processes of the type, $\eta N_1 \to \pi N_1$ followed by $\pi N_2 \to \eta N_2$ or $\pi N_2 \to \pi N_2$ followed by $\pi N_2 \to \eta N_2$ etc. These are in principle more difficult to calculate. Though this neglect cannot be justified a priori, there is reason enough to believe that these contributions may not be significantly large as argued in [16].

(iv) The ^{3}He nuclear wave function required in the calculation of the T-matrix is taken to be of the Gaussian form. The use of this wave function for the present work seems adequate in the light of the calculations performed in [13] where searches for eta-mesic states using a more sophisticated wave function for ^{3}He resulted in almost the same locations of the T-matrix poles as found using the Gaussian one.

The T-matrix for ηA elastic scattering, $t_{\eta A}$, is related to the S-matrix as,

$$S = 1 - \frac{\mu\, i\, k}{\pi}\, t_{\eta A}\,, \tag{9}$$

where k is the momentum in the ηA centre of mass system and hence, the dimensionless T-matrix required in the evaluation of time delay is given as, $T = -\left(\mu\, k /\, 2\,\pi\right) t_{\eta A}$. and used to evaluate the time delay given in (2) for the reaction $\eta\,^3\text{He} \to \eta\,^3\text{He}$. We evaluate $\Delta t_{ii}(E)$ at both positive and negative values of E in order to search for resonances and unstable bound states. As explained above with the example of the deuteron, time delay at negative energies is a valid concept and is useful in identifying the bound and unstable bound states. However, in contrast to positive energy resonances where one can "measure" the time delay from phase shifts in a scattering experiment, the time delay at negative energies is not measurable in an elastic scattering experiment. At negative energies however, it manifests in an off-shell elastic scattering, where the

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transition matrix $t_{\eta A}$ is calculated at purely imaginary momenta ($k = k' \to ik$) corresponding to $E = -k^2/2\mu$ with μ being the reduced mass of the ηA system. Physically, this would correspond to the binding of subthreshold produced η's in nuclear reactions through off shell scattering.

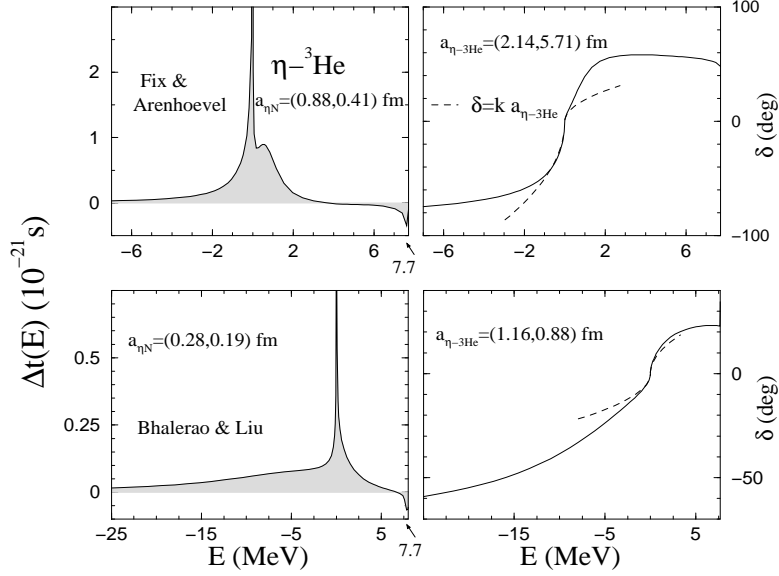Figure 2: Time delay in the elastic scattering reaction, $\eta\,^3\mathrm{He} \to \eta\,^3\mathrm{He}$, as a function of energy, $E = E_{\eta\,^3He} - m_\eta - m_{^3He}$, where $E_{\eta\,^3He}$ is the total energy available in the $\eta\,^3$He centre of mass. The vertical axis scale has been broken to display the structure of the time delay plot clearly.

In Fig. 2, we show the $\eta\,^3$He time delay plots with two different inputs for the ηN interaction [4, 14]. In [14] a coupled channel t-matrix including the πN and ηN channels with the S_{11} - ηN interaction playing a dominant role was constructed. It consisted of the meson - N* vertices and the N* propagator as given below:

$$t_{\eta N \to \eta N}(k', k; z) = \frac{g_{N^*}\beta^2}{(k'^2 + \beta^2)} \, \tau_{N^*}(z) \, \frac{g_{N^*}\beta^2}{(k^2 + \beta^2)} \tag{10}$$

with, $\tau_{N^*}(z) = (z - M_0 - \Sigma_\pi(z) - \Sigma_\eta(z) + i\epsilon)^{-1}$, where $\Sigma_\alpha(z)$ $(\alpha = \pi, \eta)$ are the self energy contributions from the πN and ηN loops. The parameters were fitted such that they reproduced scattering lengths of $a_{\eta N} = (0.75, 0.27)$ fm and $a_{\eta N} = (0.88, 0.41)$ fm, which are in agreement with values obtained from modern analyses of recent data. We chose the parameter set with $g_{N^*} = 2.13, \beta = 13\,\mathrm{fm}^{-1}$ and $M_0 = 1656\,\mathrm{MeV}$, which gave rise to $a_{\eta N} = (0.88, 0.41)$ fm. In [4], the πN, ηN and $\pi\Delta$ $(\pi\pi N)$ channels were treated in a coupled channel

formalism (so that an additional self-energy term appears in the propagator $\tau_{N^*}(z)$). The parameters of this model are, $g_{N^*} = 0.616$, $\beta = 2.36\,\text{fm}^{-1}$ and $M_0 = 1608.1\,\text{MeV}$. These parameters were obtained from a fit to the P_{33} and S_{11} phase shifts in πN elastic scattering and the differential $\pi N \rightarrow \eta N$ cross sections predicted by the model were in good agreement with data. The input parameters of this model, however, give rise to a much smaller scattering length, namely, $a_{\eta N} = (0.28, 0.19)$ fm. It should be noted that though the two models predict somewhat different total $\pi N \rightarrow \eta N$ cross sections, they both agree with data since the error bars on the data are large.

In both cases we find a peak centered slightly below zero energy. In the case of the larger ηN scattering length [14], we find an additional bump located at a positive energy of 0.5 MeV and overlapping the peak very close to threshold. In the case of the smaller $a_{\eta N}$ (lower half), there is a hint of a broad hump around -5 MeV. These findings seem to be in agreement with experiment where a binding energy of -4.4 ± 4.2 MeV for the η-mesic ^{3}He and a resonance like structure just above production threshold was reported.

Before proceeding further, we would like to mention that in the case of s-wave scattering near the elastic threshold, i.e. at zero energy, one needs to be careful in drawing conclusions from time delay plots. Using $S = exp(2i\delta)$ and comparing it with (9),

$$\delta = \frac{1}{2i}\ln(1 - \frac{i\mu k}{\pi}t_{\eta A}) = \frac{1}{2i}\ln(1 + 2ikf) \qquad (11)$$

where f is the scattering amplitude. For small k, $\delta \simeq kf$ and the behaviour of $d\delta/dE$ (the real part of which is essentially the time delay) is determined by the simple pole at $k = 0$ (or $E_{\eta^3 He} = E_{threshold}$) and the energy dependence of the scattering amplitude f. In the absence of a resonance, as $k \rightarrow 0$, $\delta = ka$, where $a = a_R + ia_I$ is the complex scattering length. For positive energies, $\Re e\delta = ka_R$, whereas for energies below zero, $k \rightarrow ik$ and $\Re e\delta = -ka_I$. In such a situation, $\Re e(d\delta/dE)$ exhibits a sharp peak at $E_{\eta^3 He} = E_{threshold}$, the sign of which is determined by the sign of the scattering length. On the other hand, if the scattering amplitude has a resonant behaviour near threshold, one would see a superposition of the two behaviours. For a state far from threshold, the rise in time delay near threshold is extremely sharp and the shape of the state remains completely unaffected. A state reasonably close to threshold is distorted in shape and one very close manifests simply by broadening the threshold singularity. In Fig. 2, we display the phase shifts and their threshold behaviour (dashed lines) which depend on the $\eta-^3$He scattering lengths. These phase shifts do display the characteristic resonant behaviour apart from their agreement with $\delta = ka$ close to threshold.

The formalism of the present work forces the three nucleon system to be in the bound state always and hence searching for unstable states beyond the ^{3}He break up threshold of 7.7 MeV as well as below -7.7 MeV is beyond the reach of the present approach. This limitation taken along with the fact that virtual target excitations could be important below the break up threshold add some uncertainty to the bump around -5 MeV. However, this bump is quite broad

and spreads over to the positive energy side near threshold where the FRA is reliable. Besides, we note that the ηN scattering length in this case is very small ($a_{\eta N} = (0.28, 0.19)$ fm) and in a comparison of the Alt-Grassberger-Sandhas (AGS) equations (which include the target excitations) with FRA, the authors in [17] found that the FRA works reasonably well for real parts of $a_{\eta N} < 0.5$ fm. The above peak is also expected to be distorted by the singularity in the threshold region as mentioned above. Thus, one cannot be sure about the exact peak position of this state, but the plot does seem to indicate the existence of one broad state in this region within the eta-nucleon model of Bhalerao and Liu.

Another interesting feature in these time delay plots is the occurrence of negative time delay around 7.7 MeV which is essentially the binding energy of the ^{3}He nucleus and is also an input to our calculation (see Eq. 4). It can be understood by noting the connection of the density of states with the energy derivative of the phase shift which also defines time delay. According to the Beth-Uhlenbeck formula [18], the difference in the density of states with and without interaction is given by the energy derivative of the phase shift. This means that when the density of states due to interaction is less than that without interaction, the energy derivative of the phase shift would be negative. The reduction in the density of states due to interaction in our context corresponds to a loss of flux from the elastic channel due to the opening of an inelastic channel (more examples in [19]).

In Fig. 3, we show the time delay plots and corresponding phase shifts using a simple separable form of the off-shell t-matrix [15]. Though this t-matrix does not contain the self-energy contributions as in [4, 14], it has the advantage that it is parametrized with the latest Crystal Ball data on $\pi N \to \eta N$ [20] and also includes the $\gamma N \to \eta N$ data [21]. In Fig. 3a, we observe subthreshold peaks for two sets of t-matrix solutions (Set A and B from [15]), out of which the solid line indicates their best solution and dashed one a similar solution without pion beam momentum correction as explained in [15]. In both these cases, however, we observe negative time delay above threshold which could be due to a repulsive interaction [10, 19]. In Fig. 3b, one sees a broad peak at threshold, very similar to those in Fig. 2. This result corresponds to Set D in [15] which is an unconventional solution obtained when the recent data from [20] is omitted. Finally, Fig. 3c displays two distinct peaks using the Set E solution which omits the data from [21]. Though we have listed only the scattering lengths corresponding to the various solutions, it should be noted that the results are also sensitive to the effective range parameter r_0 given in Table III of [15].

Before concluding, we mention an analysis [22] where, using optical potentials, the authors investigated the relation between the η-^{3}He binding energy, width and the complex scattering length, $a_{\eta-^3He}$. The strength of the potential was varied to check for which scattering lengths one can find bound states. They found that if an η-mesic ^{3}He state exists, its binding energy (width) should not exceed 5 MeV (10 MeV). It is interesting to note that though the approach of the present work which involves the indentification of the quasi bound state from positive peaks in time delay is entirely different from that in [22], the possible
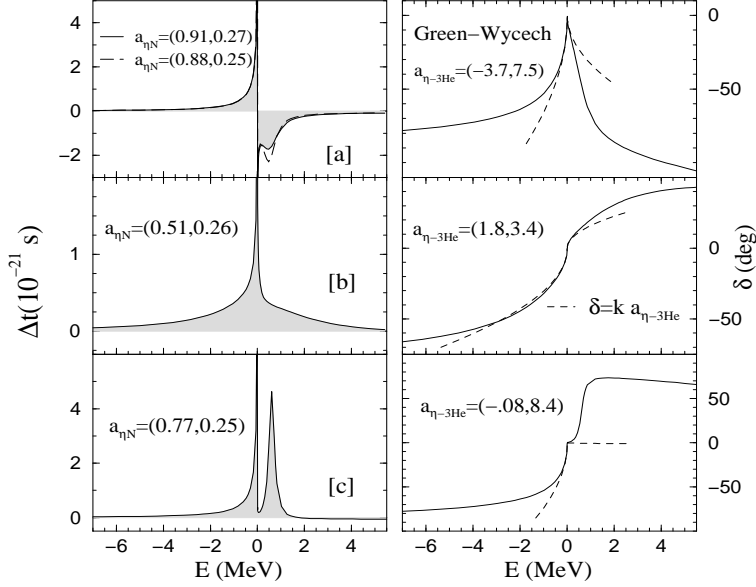
Figure 3: Same as Fig. 2, but with a different model of the ηN interaction. Scattering lengths are given in fm.

η-mesic states of the present work lie within this limit. The constraints on the energy and width in [22] were found following a systematic analysis of the η-^{3}He scattering length [23]. The values of $a_{\eta-^3He}$ obtained in the present work are outputs of the few body calculations for a given ηN interaction that enters the η-nucleus t-matrix as an input. They are listed in Figs 2 and 3. We refer the reader to [23] and the references therein for a detailed discussion on the issue of the η-nucleus scattering lengths.

In conclusion, we have investigated the possible existence of η-mesic ^{3}He using the physical concept of time delay and within the limitations of the present approach, find evidence for the existence of such states near threshold using two off-shell isobar models ([4] and [14]) of the ηN interaction and another K-matrix semiphenomenological analysis of relevant data [15]. With the availability of more accurate data on η-mesic nuclei, the present results will be useful in obtaining a better knowledge of the ηN interaction.

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